

**FAR
BEYOND**

MAT122

Logarithmic Function



Stony Brook University

Logarithmic Intro

Fact: logarithms and exponents are inverses of each other
use one to cancel other and/or solve

$$x = b^y \Leftrightarrow y = \log_b x$$

To convert between logarithm and exponential format:

1. keep base the same
2. swap x and y

ex. Write in equivalent exponential form:

- $2 = \log_5 x$

$$y = \log_b x \Rightarrow b^y = x$$

$$5^2 = x$$

$$x = 25$$

- $3 = \log_b 64$

$$b^3 = 64$$

- $y = \log_3 7$

$$3^y = 7$$

ex. Write in equivalent logarithmic form:

- $12^2 = x$

$$\log_{12} x = 2$$

- $b^3 = 8$

$$\log_b 8 = 3$$

- $e^y = 9$

$$\log_e 9 = y$$

write as

natural logarithm

$$\ln 9 = y$$

- $10^y = 7$

$$\log_{10} 7 = y$$

write as

common logarithm

$$\log 7 = y$$

Special Logarithmic Properties

$$\log_b b = 1 \quad \text{since } b^1 = b$$

ex. Evaluate $\log_7 7 = 1$

$$\log_b 1 = 0 \quad \text{since } b^0 = 1$$

ex. Evaluate $\log_5 1 = 0$

also...

$$\ln e = 1$$

$$\ln 1 = 0$$

Inverse Properties:

$$(f \circ g)(x) = x$$

$$\log_b b^x = x$$

ex. Evaluate ~~\log_7~~ $7^5 = 5$

$$(g \circ f)(x) = x$$

$$b^{\log_b x} = x$$

ex. Evaluate ~~6^{\log_6}~~ $9 = 9$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

Logarithmic Graph

Log graph is inverse of the exponential graph
with correlating base.

$$\underline{b^x}$$

domain: \mathbb{R}

range: $(0, \infty)$

asymptote: $y = 0$

$$\underline{\log_b x}$$

domain: $(0, \infty)$

range: \mathbb{R}

asymptote: $x = 0$

Logarithm Rules

More complex logarithms can be expressed in one of two formats:

- *condensed*
- *expanded*

Use these rules to help evaluate equations with logs.

Condensed Format

Product Rule

$$\log_b(MN)$$

multiplication inside parentheses

Quotient Rule

$$\log_b\left(\frac{M}{N}\right)$$

division inside parentheses

Power Rule

$$\log_b M^P$$

variable has an exponent

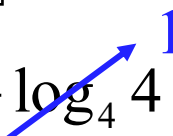
Product Rule

$$\text{Product Rule: } \log_b(MN) = \log_b(M) + \log_b(N)$$

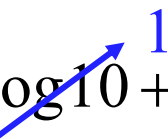
condensedexpanded
formatformat

ex. expand $\ln(xy) = \boxed{\ln x + \ln y}$

ex. expand $\log_4(7 \cdot 4) = \log_4 7 + \log_4 4 = \boxed{\log_4 7 + 1}$



ex. expand $\log(10z) = \log 10 + \log z = \boxed{1 + \log z}$



$$\log 10 = \log_{10} 10$$

ex. condense $\log_3 p + \log_3 q = \boxed{\log_3(pq)}$

ex. condense $\log_4 z + \log_4 y^7 + \log_4 5 = \boxed{\log_4(5y^7z)}$

Quotient Rule

$$\text{Quotient Rule: } \log_b \left(\frac{M}{N} \right) = \log_b(M) - \log_b(N)$$

ex. expand $\log_7 \left(\frac{19}{x} \right) = \log_7 19 - \log_7 x$
doesn't
simplify

ex. expand $\ln \left(\frac{e^3}{7} \right) = \ln e^3 - \ln 7 = 3 - \ln 7$
cancels

ex. condense $\log_3 x^4 - \log_3 \sqrt{y} = \log_3 \left(\frac{x^4}{\sqrt{y}} \right)$

Power Rule

Power rule allows the exponent of the variable to become the log's coefficient and vice versa.

$$\text{Power Rule: } \log_b M^P = P \log_b (M)$$

note:

$$(\log_b M)^P \neq \log_b M^P$$

ex. expand $\ln x^2 = 2 \ln x$

ex. expand $\log_5 7^4 = 4 \log_5 7$

ex. expand $\ln 3^x = x \ln 3$

ex. expand $\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$

$$\sqrt[n]{b} = b^{1/n}$$

Expand Logarithmic Expressions

ex. Expand the following as much as possible:

$$\bullet \log_b (x^2 \sqrt{y}) = \log_b x^2 + \log_b \sqrt{y}$$

PRODUCT

$$= 2 \log_b x + \log_b y^{1/2}$$

$$= 2 \log_b x + \frac{1}{2} \log_b y$$

$$\bullet \log_6 \left(\frac{\sqrt[3]{x}}{36y^4} \right) = \log_6 \sqrt[3]{x} - \log_6 (36y^4)$$

PRODUCT

QUOTIENT

$$= \log_6 x^{1/3} - (\log_6 36 + \log_6 y^4)$$

$$= \frac{1}{3} \log_6 x - 2 - 4 \log_6 y$$

$$\log_6 36 = 2$$

$$6^2 = 36$$

$$y = 2$$

$$\log_b (MN) = \log_b (M) + \log_b (N)$$

$$\log_b \left(\frac{M}{N} \right) = \log_b (M) - \log_b (N)$$

$$\log_b M^P = P \log_b (M)$$

Do: Expand (and evaluate where applicable):

$$\log_5 \left(\frac{\sqrt{x}}{125y^3} \right)$$

$$= \frac{1}{2} \log_5 x - 3 - 3 \log_5 y$$

Condense Logarithmic Expressions

ex. Write the following as a single logarithm:

$$\bullet \log(4x-3) + \log x = \boxed{\log(x(4x-3))}$$

PRODUCT

$$\bullet 3\ln(x+7) - 4\ln x = \ln(x+7)^3 - \ln x^4$$

2. QUOTIENT

1. POWER*

$$= \boxed{\ln \frac{(x+7)^3}{x^4}}$$

*coefficients must be 1 to condense

$$\bullet 4\log_b x + 2\log_b 6 - \frac{1}{2}\log_b y$$

$$= \log_b x^4 + \log_b 6^2 - \log_b y^{1/2}$$

PRODUCT

$$= \log_b (36x^4) - \log_b \sqrt{y}$$

QUOTIENT

convert exponents to radicals in condense

$$= \boxed{\log_b \left(\frac{36x^4}{\sqrt{y}} \right)}$$

$$\log_b(MN) = \log_b(M) + \log_b(N)$$

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

$$\log_b M^P = P\log_b(M)$$