FAR
BEYOND

MAT122

Logarithmic Function

Stony Brook University

Logarithmic Intro

Fact: logarithms and exponents are inverses of each other

use one to cancel other and/or solve

$$\boxed{x = b^y \Leftrightarrow y = \log_b x}$$

To convert between logarithm and exponential format:

1. keep base the same
2. swap x and y

ex. Write in equivalent exponential form:

- $2 = \log_5 x$

  $y = \log_b x \Rightarrow b^y = x$

  $5^2 = x$ $\boxed{x = 25}$

- $3 = \log_b 64$ $\boxed{b^3 = 64}$
- $y = \log_3 7$ $\boxed{3^y = 7}$

ex. Write in equivalent logarithmic form:

- $12^2 = x$ $\boxed{\log_{12} x = 2}$
- $b^3 = 8$ $\boxed{\log_b 8 = 3}$
- $e^y = 9$ $\log_e 9 = y$ write as $\boxed{\ln 9 = y}$ natural logarithm
- $10^y = 7$ $\log_{10} 7 = y$ write as $\boxed{\log 7 = y}$ common logarithm

Special Logarithmic Properties

$\log_b b = 1$ since $b^1 = b$

$\log_b 1 = 0$ since $b^0 = 1$

ex. Evaluate $\log_7 7 = 1$

ex. Evaluate $\log_5 1 = 0$

also…

$\ln e = 1$

$\ln 1 = 0$

Inverse Properties:

$(f \circ g)(x) = x$
$(g \circ f)(x) = x$

$\log_b b^x = x$

$b^{\log_b x} = x$

ex. Evaluate $\cancel{\log_7 7}^{5} = 5$

ex. Evaluate $\cancel{6^{\log_6}}{}^{9} \quad = 9$

$\ln e^x = x$

$e^{\ln x} = x$

Logarithmic Graph

Log graph is inverse of the exponential graph with correlating base.

b^x	$\log_b x$
domain: $\mathbb{R}$	domain: $(0,\infty)$
range: $(0,\infty)$	range: $\mathbb{R}$
asymptote: $y=0$	asymptote: $x=0$

Logarithm Rules

More complex logarithms can be expressed in one of two formats:

- *condensed*
- *expanded*

Use these rules to help evaluate equations with logs.

Condensed Format

Product Rule $\log_b (MN)$ multiplication inside parentheses

Quotient Rule $\log_b \left(\frac{M}{N} \right)$ division inside parentheses

Power Rule $\log_b M^P$ variable has an exponent

Product Rule

Product Rule: $\log_b(MN) = \log_b(M) + \log_b(N)$

condensed format — expanded format

ex. expand $\ln(xy) = \boxed{\ln x + \ln y}$

ex. expand $\log_4(7 \cdot 4) = \log_4 7 + \cancelto{1}{\log_4 4} = \boxed{\log_4 7 + 1}$

ex. expand $\log(10z) = \cancelto{1}{\log 10} + \log z = \boxed{1 + \log z}$

$\log 10 = \log_{10} 10$

ex. condense $\log_3 p + \log_3 q = \boxed{\log_3(pq)}$

ex. condense $\log_4 z + \log_4 y^7 + \log_4 5 = \boxed{\log_4\left(5y^7z\right)}$

Quotient Rule

Quotient Rule: $\log_b\left(\dfrac{M}{N}\right) = \log_b(M) - \log_b(N)$

ex. expand $\log_7\left(\dfrac{19}{x}\right) = \boxed{\log_7 19 - \log_7 x}$

doesn't simplify

ex. expand $\ln\left(\dfrac{e^3}{7}\right) = \cancel{\ln e}^{3} - \ln 7 = \boxed{3 - \ln 7}$

cancels

ex. condense $\log_3 x^4 - \log_3 \sqrt{y} = \boxed{\log_3\left(\dfrac{x^4}{\sqrt{y}}\right)}$

Power Rule

Power rule allows the exponent of the variable to become the log's coefficient and vice versa.

Power Rule: $\log_b M^P = P\log_b(M)$

note:
$\left(\log_b M\right)^P \neq \log_b M^P$

ex. expand $\ln x^2 = \boxed{2\ln x}$

ex. expand $\log_5 7^4 = \boxed{4\log_5 7}$

ex. expand $\ln 3^x = \boxed{x\ln 3}$

ex. expand $\ln\sqrt{x} = \ln x^{1/2} = \boxed{\frac{1}{2}\ln x}$

$\sqrt[n]{b} = b^{1/n}$

Expand Logarithmic Expressions

ex. Expand the following as much as possible:

- $\log_b\left(x^2\sqrt{y}\right) = \log_b x^2 + \log_b \sqrt{y}$

  PRODUCT

  $= 2\log_b x + \log_b y^{1/2}$

  $= \boxed{2\log_b x + \frac{1}{2}\log_b y}$

- $\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right) = \log_6 \sqrt[3]{x} - \log_6(36y^4)$

  QUOTIENT

  PRODUCT

  $= \log_6 x^{1/3} - (\log_6 36 + \log_6 y^4)$

  $= \boxed{\frac{1}{3}\log_6 x - \quad 2 \quad - 4\log_6 y}$

  $\log_6 36 = y$

  $6^y = 36$

  $y = 2$

$$\boxed{\begin{aligned} \log_b(MN) &= \log_b(M) + \log_b(N) \\ \log_b\left(\frac{M}{N}\right) &= \log_b(M) - \log_b(N) \\ \log_b M^P &= P\log_b(M) \end{aligned}}$$

Do: Expand (and evaluate where applicable):

$$\log_5\left(\frac{\sqrt{x}}{125y^3}\right)$$

$$= \boxed{\frac{1}{2}\log_5 x - \quad 3 \quad -3\log_5 y}$$

Condense Logarithmic Expressions

$$\log_b(MN) = \log_b(M)+\log_b(N)$$

$$\log_b\left(\frac{M}{N}\right) = \log_b(M)-\log_b(N)$$

$$\log_b M^P = P\log_b(M)$$

ex. Write the following as a single logarithm:

- $\log(4x-3)\enclose{circle}{+}\log x \;=\boxed{\log\left(x(4x-3)\right)}$

PRODUCT

- $3\ln(x+7)\enclose{circle}{-}4\ln x = \ln(x+7)^3-\ln x^4$ 2. QUOTIENT

1. POWER*

$$=\boxed{\ln\frac{(x+7)^3}{x^4}}$$

*coefficients must be 1 to condense

- $4\log_b x+2\log_b 6-\frac{1}{2}\log_b y$

$$=\underbrace{\log_b x^4+\log_b 6^2}_{\text{PRODUCT}}-\log_b y^{1/2}$$

convert exponents to radicals in condense

↓

$$=\underbrace{\log_b(36x^4)\qquad-\log_b\sqrt{y}}_{\text{QUOTIENT}}\quad=\boxed{\log_b\left(\frac{36x^4}{\sqrt{y}}\right)}$$